Calc BC Solns Parametric Practice

① $x = 4\sin t \quad y = 2\cos t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \; ; \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(-\frac{1}{2}\tan t\right)}{4\cos t}$$

$$= \frac{-2\sin t}{4\cos t} \qquad\qquad = \frac{-\frac{1}{2}\sec^2 t}{4\cos t}$$

$$= -\frac{1}{2}\tan t \qquad\qquad = -\frac{1}{8\cos^3 t}$$

② $x = t^2 - 3t \quad y = t^3$

$$\frac{dy}{dx} = \frac{3t^2}{2t-3} \; ; \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{3t^2}{2t-3}\right)}{2t-3}$$

$$= \frac{6t(2t-3) - 2(3t^2)}{(2t-3)^3}$$

$$= \frac{6t^2 - 18t}{(2t-3)^3}$$

③ $x = \frac{1}{t} \quad y = -2 + \ln t$

$$\frac{dy}{dx} = \frac{\frac{1}{t}}{-\frac{1}{t^2}} \; ; \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(-t)}{-\frac{1}{t^2}}$$

$$= -t \qquad\qquad = \frac{-1}{-\frac{1}{t^2}}$$

$$= t^2$$

④ $x = 2 + \cos t$ $\qquad y = -1 + \sin t$

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t}$$

$$= -\cot t$$

horizontal: $\frac{dy}{dt} = 0$; $\cos t = 0$

$t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{2} \Rightarrow (2, 0)$

$t = \frac{3\pi}{2} \Rightarrow (2, -2)$

vertical: $\frac{dx}{dt} = 0$

$-\sin t = 0$

$t = 0, \pi, 2\pi$

$t = 0 \Rightarrow (3, -1)$

$t = \pi \Rightarrow (1, -1)$

⑤ $x = \sec t$ $\qquad y = \tan t$

$$\frac{dy}{dx} = \frac{\sec^2 t}{\sec t \cdot \tan t} = \frac{\sec t}{\tan t}$$

horizontal: $\frac{dy}{dt} = 0$; $\sec t = 0$

<u>No horizontal tangents</u>

vertical: $\frac{dx}{dt} = 0$; $\tan t = 0$

$t = 0, \pi, 2\pi$

$t = 0 \Rightarrow (1, 0)$

$t = \pi \Rightarrow (-1, 0)$

⑥ $$L = \int_0^1 \sqrt{(t^2)^2 + (t)^2}\ dt$$

$$= \int_0^1 \sqrt{t^4 + t^2}\ dt$$

$$= \int_0^1 \sqrt{t^2(t^2+1)}\ dt$$

$$= \int_0^1 t\sqrt{t^2+1}\ dt$$

$u = t^2+1$
$du = 2t\ dt$
$u(1) = 2$
$u(0) = 1$

$$= \frac{1}{2}\int_1^2 u^{1/2} = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^2 = \frac{1}{3}\left[\sqrt{8} - 1\right]$$

⑦ $x = e^t - t^2 \qquad y = t + e^{-t}$

$$\frac{dx}{dt} = e^t - 2t \qquad \frac{dy}{dt} = 1 - e^{-t}$$

$$L = \int_{-1}^{2} \sqrt{(e^t - 2t)^2 + (1 - e^{-t})^2}\ dt$$

$$= 4.497$$